A comparison analysis of two alternative dairy cattle replacement strategies: Optimization *versus* Simulation models.

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Abstract

Kalantari, A.S., Cabrera, V.E., Solis, D. 2014. A comparison analysis of two alternative dairy cattle replacement strategies: Optimization versus Simulation models. The objective of this study was to compare the optimal replacement decisions using two alternative state-of-the-art models: the optimization dynamic programming model and the Markov chain simulation model. Lactation, month in milk and pregnancy status were used to describe cow states in a herd in both models. Both models were fed with the same parameters and transition probabilities to make the fairest comparison possible. The cow value calculated by the Markov chain model was compared against the retention pay-off estimated by the dynamic programming model. These values were used to rank all the animals in the herd. Then, the rank correlation (Spearman's correlation) was calculated between results of both models. The overall correlation was 95%, which showed a strong linear relationship between rankings of animals from the two models. Moreover, the lowest 10% ranking cows -which are the most likely replacement candidates- displayed a greater correlation, 98%. Thus, the final replacement decisions with both models were similar. A post optimality analysis was used to explore the effect of the optimal replacement decisions on the herd dynamics and herd net return. The results showed a comparable herd structure by both models. A net return was improved US$6/cow per year by using replacement decisions of both dynamic programming model and the Markov chain cow value model.

Keywords: herd economics, optimization, replacement policy, simulation

INTRODUCTION

The ability of farmers to make right decisions at the right times significantly determines the success of any enterprise. This success can be stated as maximizing profit. It has been shown that total profit is highly affected by replacement decisions (van Arendonk, 1984), and reproductive performance (Britt, 1985). Reproductive performance attained special attention in the literature (Olynk and Wolf, 2009; Cabrera and Giordano, 2010; Giordano *et al*., 2011; Giordano *et al*., 2012) as a result of its prominent economic impact on the profitability of dairy farms.

Over the past decades several studies have analyzed the optimum replacement interval in dairy herds and factors that affect these decisions (Smith, 1973; van Arendonk, 1985; Kristensen, 1988; De Vries, 2004, Groenendaal *et al*., 2004; Demeter *et al*., 2011; Cabrera, 2012). Simultaneous accounting of several biological and economic parameters is necessary to determine the optimum time of replacing a cow. Milk production level, pregnancy, stage of lactation, parity and transition probabilities such as involuntary culling, pregnancy, and abortion are considered the most important factors affecting replacement decisions (Kalantari *et al*., 2010). Alternative approaches have been proposed to handle these factors and find the optimum replacement strategy including marginal net revenue (MNR) (van Arendonk, 1984; Groenendaal *et al*., 2004), dynamic programming (DP) (Smith, 1973; van Arendonk, 1985; De Vries, 2004), and stochastic simulation models (Marsh *et al*., 1987; Dijkhuizen and Stelwagen, 1988; Kristensen and Thysen, 1991). The first two methods are based on the production function approach in which the cow's revenue and costs are modeled during cow's lifetime (Groenendaal *et al*., 2004). The limitation of MNR is its inability to include the variation in expected milk production of the present cow and subsequent replacement heifers, and the genetic gain of replacement heifers (Groenendaal *et al*., 2004). The DP technique overcomes both of these limitations. However, because its complexity, the usage of DP models has been restricted to research analysis and not for building decision support systems for practical decision-making and farm management. The Monte Carlo stochastic simulation approach has been used to calculate the total expected net returns during next year and that value was used for ranking animals. Kristensen and Thysen (1991) compared the decisions being made by DP and stochastic simulation and reported insignificant difference between the two models.

Recently, Cabrera (2012) used a Markov chain simulation model to find a suboptimal replacement strategy. In brief, this method calculates the net present value for a cow and its potential replacement, which could be used to decide whether to keep or replace a dairy cow. This method does not have the complexity of DP models and overcomes the limitation of MNR method because it can include expected variations in the cow and replacement performances. Cabrera (2012) reported that trend and replacement strategies found with the newly Markov chain model would be similar to those found with DP models. However, such study did not include a formal comparison with a DP model. Consequently, the objectives of this study are to compare the replacement decision strategies reached with a DP and a

Markov chain model; and to compare the effect of optimal replacement strategy on the herd structure and net revenue.

MATERIALS AND METHODS

In this study we compare the outcomes of two alternative models currently used in the literature to offer dairy cattle replacement strategies. These models are: the DP model, adapted from Kalantari *et al*. (2010); and the Markov chain model from Cabrera (2012). Both models were set to follow similar specifications and parameters.

Three state variables were used to describe cows in both models. Cow states were defined by lactation number (l = 1 to 10), month in milk (m = 1 to 20), and month in pregnancy (p = 0 to 9; 0 for open cow and 1 to 9 for pregnant). After discounting impossible states, each model had 1,000 possible states. There states were also the number of common stochastic elements for transition probabilities such as the probability of abortion, pregnancy, and involuntary culling. These transition probabilities were used to define the flow process of cows among states from one month to another. For example, an open cow could become pregnant in the current month or be involuntary culled (retired because the cow can no longer produce) in next month according to these probabilities (Cabrera, 2012).

Although both models rely on Markov chains as their underline structure, they have different control mechanisms. The transition probability matrix is the only governing rule that changes states from one stage to another in a Markov chain model. However, there is an extra step at each stage on the DP model, which is to select the optimal action in the current stage for the specified state variables. In other words, the addition of a system control mechanism, which can be defined with the term Markov decision process instead of Markov chain (Gosavi, 2003).

The DP model used the 'divide and conquer' algorithm to break the multi-stage problem into a series of independent single-stage problems. The objective function was to maximize the net present value of revenues from the current cow and its potential replacements (Kalantari *et al*., 2010). The objective function can be shown in terms of mathematical notion as follows:

$$F_{l,m,p} = Max\left[Keep_{l,m,p}, Repl_{l,m,p}\right] \quad (1)$$

Where $\text{Keep}_{l,m,p}$ = expected net present value **(NPV)** of keeping the cow in lactation (l), month in milk (m), and pregnancy (p), given the optimal decisions in the remainder stages and $\text{Repl}_{l,m,p}$ = expected net present value of replacing the cow given the optimal decisions in the remainder stages. The detailed formulation of calculating the keep and replace values for different states can be found in Kalantari *et al*. (2010). Retention pay-off (**RPO**), which is the expected profit from keeping the cow compared with immediate replacement (De Vries, 2004), was calculated using the following equation:

$$RPO_{l,m,p} = Keep_{l,m,p} - Repl_{l,m,p} \quad (2)$$

The RPO represents the value of a given cow (represented by l,m,p). The RPO can take positive, zero, or negative values. A positive RPO determines that keeping the cow for another month has a higher net return than replacing it, whereas negative RPO means that immediate replacement has a higher net return than keeping the cow. The RPO can be used to rank all cows in the herd to find out the cows that are most likely replacement candidates.

A Markov chain model with monthly stage was developed to predict the herd structure at each stage following Cabrera (2012). The NPV of the cow and its replacement is calculated at each stage until the model reaches the condition of 'steady state'. Steady state is achieved when the proportion of cows in all states remain constant in two subsequent stages. Steady state in the model defined here always occurred before iteration number 150th (which is the same as 150 months in the future). Formulas for calculating the proportion of cows at each stage are described in detail in Cabrera (2012).

The NPV of the cow and its replacement were calculated by adding all economic values at each stage from the start of simulation until a time when the model was at steady state. Economic values at each stage were calculated as the sum product of the net revenue of each state and the corresponding herd structure. The formula, following notations in Cabrera (2012), for this calculation follows:

$$NPV = \sum_{i=1}^{150} \partial \left[\sum_{l=1}^{10} \sum_{m=1}^{25} \sum_{p=0}^{9} (Mi - Fc + Ci - NRCc - Mc - RCc - Rc)_{l,m,p} \times (COW)_{l,m,p} \right]_i \qquad (3)$$

Where ∂ is interest rate, Mi milk income, Fc feed cost, Ci calf income, NRCc non-reproductive culling cost, Mc Mortality cost, RCc reproductive culling cost, Rc reproductive cost, and COW the proportion of cows (herd structure) at each stage (i) for given state variable (represented by *l,m,p*). After finding the NPV for both the cow and its replacement the cow value was estimated by using the following equation:

Cow Value = NPV Cow – NPV Replacement – (Replacement Cost – Salvage Value – Calf Value) (4)

This cow value could then be used for deciding whether to keep or replace a cow based upon the sign of the value. Positive cow value (like positive RPO) means that the cow would bring more net revenue than its replacement and therefore the best decision would be to keep the cow. A negative cow value means that replacement is more profitable than keeping it.

i) Milk production: The MilkBot function (Ehrlich, 2011) was used to fit milk production curves for the first, second and third and later lactations. The MilkBot predicts milk yields, Y(m), as a function of time after parturition or months in milk, m. Four parameters, a (scale), b (ramp), c (offset), and d (decay), control the shape of the lactation curves (Ehrlich, 2011).

$$Y(m) = a\left(1 - \frac{e^{\frac{c-m}{b}}}{2}\right)e^{-d \times m} \qquad (5)$$

Using this function the 305 day estimated milk production (kg) were approximately

10,000, 11,000 and 12,000 for the first 3 lactations, respectively.

ii) Live body weight: Average monthly live weight for each state was calculated using Korver function (Korver *et al.*, 1985) as described by van Arendonk (1985). Body weights were used to calculate the carcass value of the replaced cow and to estimate dry matter intake for each cow state.

iii) Dry matter intake: Daily dry matter intake was calculated using Spartan 2 (VandeHaar et al., 1992) equation; which is a function of maintenance and milk production according to month in milk, m. This function used body weight and 4% fat corrected milk yield as inputs.

$$DMI_m = (0.02 \times BW_m) + (0.3 \times 4\%FCM_m) \quad (6)$$

Where BW is the live body weight and 4%FCM is 4% fat corrected milk.

iv) Calf value: It was assumed that all 1 week old calves are sold and the value was assumed to be the weighted average of the value for male and female calves (Meadows *et al.*, 2005).

v) Involuntary culling: Cows at every state had the risk of being involuntary culled. The risk of involuntary culling was increased by lactation and MIM. Data from De Vries *et al*. (2010) was used to incorporate these transition probabilities.

vi) Reproduction: Voluntary waiting period of 60 days (time when cows are eligible for insemination) and an 18% 21-day pregnancy rate were assumed. Cows were not bred anymore after 10 MIM (a.k.a., cut-off time). Pregnancy losses were included following De Vries (2006).

vii) Economic parameters: Replacement heifer cost was set at US$1,300/cow. Feed price for lactating and dry cows were set at US$0.22/kg and US$0.18/kg, respectively (Cabrera, 2012). Other economic variables are summarized in Table 1.

Table 1. Economic variables used for both models: dynamic programming (DP) and Markov chain (MC)

Economic variables	Value
Replacement cost, US$/cow	1,300
Carcass value, US$/kg	0.38
Calf value, US$/calf	100
Milk price, US$/kg	0.35
Feed price for lactating cow, US$/kg	0.22
Feed price for dry cows, US$/kg	0.18
Interest rate, %/year	6

viii) Computer implementation: The DP model as originally developed by Kalantari *et al*. (2010) was used to find the optimal replacement decisions. The Markov chain cow value model described by Cabrera (2012) was re-coded as a standalone executable program with Visual Basic Net 2010 (Microsoft Corp., Redmond, WA).

The most important result of these two models was the ranking of all the animals in the herd according to their expected cow value or RPO. Therefore, cow value (calculated from Markov chain model) and RPO (from DP model) were used to rank animals and compare both models' results. The Spearman's rank correlation test was used to compare rankings from both models. The "spearman" package

(Savicky, 2009) in R statistical software (R Development Core Team, 2011) was used to perform this statistical test.

After finding the optimal decisions with a DP model, Markov chain models are used to find the herd demographics (herd structure) and economic parameters under optimal decisions. Three different scenarios were desgined to compare the effect of optimal decisions on the overall herd dynamics and herd net return. The first scenario used the Markov chain model as described in Cabrera (2012). The second scenario ran the Markov chain model under the optimal decisions found out by the DP model (De Vries, 2004; Kalantari *et al*., 2010). And the third scenario used a 2-step solution procedure of the Markov chain model. Negative values in the first solution were considered replacement decisions that were applied as optimal decisions for the second solution.

Sensitivity analysis was later used to assess the effect of change of the main parameters on the accordance of the two models results. The most important factors affecting the culling decisions have been well studied and include milk production level and replacement cost (van Arendonk, 1985; van Arendonk and Dijkhuizen, 1985; Kalantari *et al*., 2010). Therefore, the effect of 20% change in milk production level and 20% change in heifer purchase price were studied.

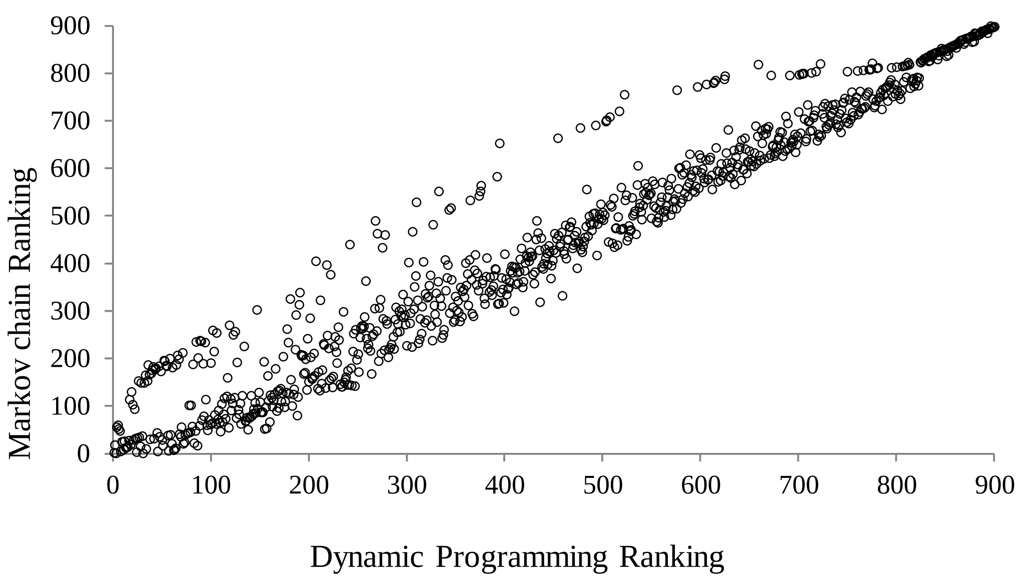
RESULTS AND DISCUSSION

We first compare the similarities between the alternative methods used in this study. The cow value ranking accrued by solving both models had a strong linear relationship. Spearman's correlation (*rho*) between rankings of the 1,000 possible states was 89% (d.f. = 998, p-value < 0.0001). This correlation factor was affected by methodological differences between models, mostly regarding to the last lactation. In DP model, cows in their last lactation and late MIM were considered to be at their end of productive life and therefore replaced regardless of their pregnancy status. The keep value for these cows was calculated with a different equation than other cow states, i.e., Equation (5) in Kalantari and Cabrera (2012), which forces replacement of these cows. In fact, this forced replacement of DP formulation affects sequentially all lactations, but has the highest impact in the last lactation, because each value is dependent on the optimal decision of the next cow state in the previous stage. However, in Markov chain model the value of the cow was calculated the same way regardless of lactation, and there was no distinction between cow value calculations of different lactations. Under those circumstances, last lactation was excluded for further analyses. After this exclusion, Spearman's correlation increased to 95% (d.f. = 898, p-value < 0.0001). The weighted average cow value -estimated by Markov chain model and weighted by proportion of cows in different states- corresponded to this ranking was US$554 and in DP model the average RPO was US$542. In both models the best ranked cow (highest positive value) was a fresh cow in third lactation - Markov chain with US$872 and DP with US$917. Also, the least valuable cow was shared by both models as a cow in 9th lactation, last month in milk, and non-pregnant. DP model's RPO for this cow was -US$44 and cow value in Markov chain was -US$355. This negative RPO or cow value means that replacing a cow would be more profitable than keeping the cow one more month in the herd.

Difference in the magnitude of the values is explained by the fact that DP follows an optimal pathway and does not accumulate negative values. However, there is no optimal strategy in the Markov chain model.

A scatter diagram of the ranking of cow values in both models for 900 states over 9 lactations is shown in Fig. 1. Rankings are closer at the beginning and at the end of the diagram. The diagram shows a bifurcation in the rankings and it is obvious that the rank for some cows does not follow the same pattern in both models. The upper groups of points in the diagram correspond to open cows in early lactation. However, these cows are far for being candidates for replacement.

Fig. 1. Ranking relationship between the dynamic programming model (DP) retention pay off (RPO) and Markov chain model (MC) cow value



From a practical decision-making and management point of views, the most important part of Fig. 1 is the end tail of the graph (right top corner) which represents the lowest ranking cow states. These cow states with the lowest values are the most likely candidates for replacement decisions. The agreement (based on *rho*) between the two models was 98% on a state space represented by 10% of all cow states in the model. The percentage of negative values in the two models was not the same, i.e., 10% of all states in the DP model (corresponding to open cows >12 MIM in the first lactation and >10 MIM in other lactations), and 12% of all states in the Markov chain model (corresponding to open cows > 10 MIM in the first lactation, > 9 MIM in the second lactation, and >8 MIM in later lactations). Since voluntary replacement decisions will not exceed 4% of the herd in one month (Fetrow *et al.*, 2006), this result indicates that final and practical replacement decisions are almost identical with both models.

Table 2 shows the breakdown of the overall correlation by pregnancy status, parity number and stage of lactation. Generally, all the correlation factors are greater than 90%, which indicates strong positive relationships between models' results. It should also be mentioned that different pregnancy status showed strong Spearman's correlation, which suggested that the models also had a high agreement based on pregnancy status.

Table 2. Spearman's correlation (*rho*) between dynamic programming model (DP) retention pay off (RPO) and Markov chain model (MC) cow value broken down by pregnancy status, parity and stage of lactation with number of pair observations from models (n) at each state.

States	rho	States	rho
Open (n=171)	0.995	3rd Parity (n=100)	0.968
1st MIP[1] (n=81)	0.970	4th Parity (n=100)	0.964
2nd MIP (n=81)	0.976	5th Parity (n=100)	0.957
3rd MIP (n=81)	0.982	6th Parity (n=100)	0.954
4th MIP (n=81)	0.989	7th Parity (n=100)	0.955
5th MIP (n=81)	0.994	8th Parity (n=100)	0.957
6th MIP (n=81)	0.992	9th Parity (n=100)	0.951
7th MIP (n=81)	0.966	Early lactation (MIM[2]=1,2) (n=18)	0.742
8th MIP (n=81)	0.881	Mid lactation (MIM=3-8) (n=243)	0.838
9th MIP (n=81)	0.916	Late lactation (MIM=9-14) (n=459)	0.978
1st Parity (n=100)	0.964	Very late lactation (MIM=15-19) (n=180)	0.995
2nd Parity (n=100)	0.973		

[1] MIP = month in pregnancy, [2] MIM = month in milk

Table 3. Economic parameters and herd structure resulting of Markov chain model simulations under different scenarios

Scenario	Economic Parameters (US$)						Herd structure						
	Net return	Milk sales	Feed cost	Calf sales	Culling cost	Reproductive cost	Parity 1 (%)	Parity 2 (%)	Parity 3 (%)	Parity ≥4 (%)	DIM (d)	Pregnant (%)	Lactating (%)
Markov chain	1,584	3,266	-1,402	63	-274	-69	34.38	25.4	16.69	23.2	138	60.8	81.22
Markov chain with DP optimal decisions	1,590	3,263	-1,401	63	-265	-69	34.84	25.26	16.59	23.04	141	60.53	81.48
Markov chain with suboptimal decisions [1]	1,590	3,279	-1,400	63	-280	-71	36.28	26.27	16.46	20.99	135	60.6	81.23

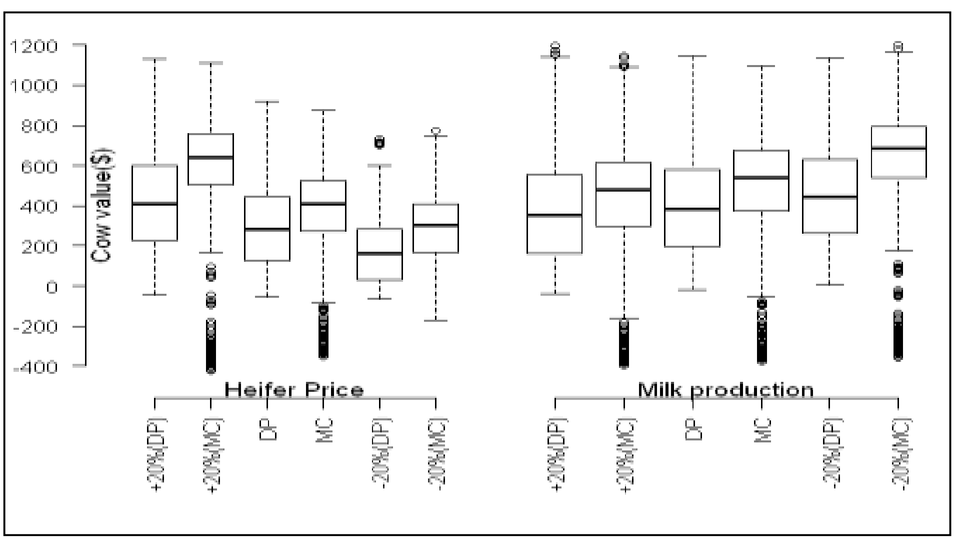
[1] 2-step solution procedure of the Markov chain model

Post optimality analyses are summarized in Table 3. The first scenario that used a Markov chain without any optimal decisions reported a net retun of US$1,584/cow per year. The net return under optimal decisions from DP was US$6/cow per year higher than the Markov chain without optimal decisions. As expected, this difference was mostly originated from reduced culling costs. Therefore, changing replacement policies according to DP results would equate in extra US$6/cow per year.

The net return resulting from Markov chain with suboptimal decisions (2-step solution scenario) was equal to the one using the DP's optimal decisions, although there were slight differences in specific economic components. Main differences between these two scenarios occurred in milk income and culling costs. Culling cost in the Markov chain model was mainly affected by applying the cut-off at 10 MIM and also having 2% more non reprodutive culling than the DP optimal decisions. Although the cut-off MIM applied equally in both models, this cut-off in the Markov chain model indicated replacement for these cows (reproductive culling). However, in DP model, cut-off MIM only meant a different calcuation of the keep value, which did not include reproductive service costs, (Kalantari and Cabrera, 2012). Another source of net return difference between the 2-step Markov chain and the DP model was higher milk sales in the Markov chain model. This difference was also related to the cut-off MIM. Cows were culled at 10 MIM in Markov chain model, which resulted in a slightly different herd structure (more early lactation cows) that yielded increased total milk revenue (Table 3).

Herd structure and dynamics at steady state of the 3 scenarios studied are also summarized in Table 3. The Markov chain and DP model's overall herd structures are not substantially different. However, results from the Markov chain under suboptimal decisions (2-step solution) showed discrepencies with results of both the original Markov chain and the DP model. The most important difference was a 1.44% change in the proportion of cows in the first parity in favor of the Markov chain with suboptimal decisions. This difference could be attributed to higher culling rates (mainly reproductive culling) in this scenario. Twenty percent changes in the milk production and heifer price did not affect the overall correlation factor of two models, remaining greater than 90% in every scenario. The effect of these changes on cow value is illustrated in Fig. 2. Because the optimal pathway is followed in DP model through iterations, no much negative values are accumulated and the minimum observed was -US$44. The dispersion of cow values in the Markov chain model was higher than in the DP (Fig. 2).

Fig. 2. Cow values (heifer price and milk production, US$) estimated using the dynamic programming model (DP) and Markov chain model (MC)



Both models have some advantages and disadvantages regarding computer programming. Despite the fact that today's computers are every time faster and more powerful, complexity of DP models in problem formulization and conceptualization (Burt, 1982) are still limiting, mostly when the main aim is to develop practical decision-making and management applications. Markov chain cow value models are easier to implement and simpler for computer programming.

Culling decisions in farms are usually for a few candidate cows for replacement, and therefore, models should evaluate the cow value just for those cows. Inability to calculate directed cow-specific RPO is another disadvantage of the DP model when thinking on computational easiness and practical decision-making. The DP model needs to calculate the keep and replace values for all the possible states in the problem within a solution. The Markov chain model can easily overcome this problem by targeting most likely replacement animals, saving great amount of computational time.

On the other hand, calculation of the value for all cow states is a major advantage of the DP model over the Markov chain cow value model. In order to evaluate and compare the results of both models in this study, assessment of all the possible states was necessary with the Markov chain model, which, at the end, took more computational time than the DP model. Nevertheless, this longer time could have been substantially reduced by using parallel programming techniques, which take advantage of multi-core processors (Ostrovsky, 2010). Parallel programming is not an option for DP because of its stepwise intrinsic nature. That is, each iteration evaluation depends on the results from the previous iteration. The new Markov chain model is a perfect candidate for running in parallel because each state evaluation is completely independent from the others.

CONCLUSIONS

We found a strong correlation (95%) in replacement decisions resulting from using two completely different modeling approaches: The classical and state-of-the-art dynamic programming framework and a newly developed technique using simple simulation of Markov chains. Post optimality analyses demonstrated that overall long-term herd structure and herd net returns resulting from models' replacement policies were very similar. These results strongly support that the newly developed Markov chain is a good alternative for practical dairy decision-making and for the development of decision support systems.

RESUMEN

El objetivo de este estudio fue comparar las decisiones de reemplazo óptimos utilizando dos modelos avanzados: el modelo de optimización de programación dinámica y el modelo de simulación de la cadena de Markov. Lactancia, mes en leche y el estado de preñez fueron utilizados para describir los estados de la vaca en un rebaño en ambos modelos. Ambos modelos fueron alimentados con los mismos parámetros y las mismas probabilidades de transición para hacer la comparación más justa posible. El valor de la vaca calculado por el modelo de cadena de Markov se comparó contra la retención de amortización calculada por el modelo de programación dinámica. Estos valores se utilizaron para clasificar a todos los animales del rebaño. Seguidamente, se calculó la correlación de rango (correlación de Spearman) entre

los resultados de ambos modelos. La correlación global fue 95%, lo que demostró una fuerte relación lineal entre la clasificación de los animales en ambos modelos. Adicionalmente, esta correlación, al considerar sólo el 10% más bajo rango de vacas -que son las candidatas más probables de reemplazo, fue aún mayor, 98%. Por lo tanto, las decisiones de reemplazo finales con ambos modelos fueron similares. Luego se utilizó un análisis post óptimo para explorar el efecto de las decisiones de reemplazo óptimos en la estructura dinámica del hato y el retorno neto. Los resultados mostraron una estructura de animales comparables en ambos modelos y el retorno neto mejoró US$ 6/cow al año con el uso de las decisiones de reemplazo com ambos modelos.

Palabras clave: Economía bovina, simulación, niveles óptimos de reemplazo.

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REFERENCES

Altman, D.G. 1991. Practical Statistics for Medical Research. Chapman & Hall, London. 611pp.

Britt, J.H. 1985. Enhanced reproduction and its economic implications. Journal of Dairy Science. 68:1585-1592.

Burt, O.R. 1982. Dynamic programming: Has its day arrived? West. Journal of Economic. 7: 381-394.

Cabrera, V.E. 2010. A large Markovian linear program to optimize replacement policies and dairy herd net income for diets and nitrogen excretion. Journal of Dairy Science. 93: 394-406.

Cabrera, V.E. and J.O. Giordano. 2010. "Economic decision making for reproduction". In Proc. Dairy Cattle Reproduction Council Convention, St. Paul, MN, p. 77-86.

Cabrera, V.E. 2012. A simple formulation and solution to the replacement problem: A practical tool to assess the economic cow value, the value of a new pregnancy, and the cost of pregnancy loss. Journal of Dairy Science. 95: 4683-4698.

Demeter, R.M., Kristensen, A. R., Dijkstra, J., Oude Lansink, A.G.J.M., Meuwissen, M.P.M., and J.A.M. van Arendonk. 2011. A multi-level hierarchic Markov process with Bayesian updating for herd optimization and simulation in dairy cattle. J. Dairy. Sci. 94, 5938-5962.

De Vries, A., 2004. Economics of delayed replacement when cow performance is seasonal. Journal of Dairy Science. 87: 2947-2958.

De Vries, A. 2006. Economic value of pregnancy in dairy cattle. Journal of Dairy Science. 89: 3876-3885.

De Vries, A., Olson, J.D. and P.J. Pinedo. 2010. Reproductive risk factors for culling and productive life in large dairy herds in the eastern United States between 2001 and 2006. Journal of Dairy Science. 93: 613-623.

Dijkhuizen, A. and J. Stelwagen. 1988. An economic comparison of four insemination and culling policies in dairy herds, by method of stochastic

simulation. Livestock Production Science. 18: 239-252.
Ehrlich, J.L. 2011. Quantifying shape of lactation curves, and benchmark curves for common dairy breeds and parities. The Bovine Practitioner. 45: 88-96.
Fetrow, J., Nordlund, K.V. and H.D. Norman. 2006. Invited review: Culling: Nomenclature, definitions, and recommendations. Journal of Dairy Science. 89: 1896-1905.
Giordano, J. O., Fricke, P.M., Wiltbank, M.C. and V.E. Cabrera. 2011. An economic decision-making support system for selection of reproductive management programs on dairy farms. Journal of Dairy Science. 94: 6216-6232.
Giordano, J.O., Kalantari, A.S., Fricke, P.M., Wiltbank, M.C. and V.E. Cabrera. 2012. A daily herd Markov-chain model to study the reproductive and economic impact of reproductive programs combining timed artificial insemination and estrus detection. Journal of Dairy Science. 95: 5442-5460.
Gosavi, A. 2003. Simulation-Based Optimization: Parametric Optimization Techniques and Reinforcement learning. Kluwer Academic Publishers., Massachusetts. 554 pp.
Groenendaal, H., Galligan, D.T. and H.A. Mulder. 2004. An economic spreadsheet model to determine optimal breeding and replacement decisions for dairy cattle. Journal of Dairy Science. 87: 2146-2157.
Kalantari, A.S., Mehrabani-Yeganeh, H., Moradi, M., Sanders, A.H. and A. De Vries. 2010. Determining the optimum replacement policy for Holstein dairy herds in Iran. Journal of Dairy Science. 93: 2262-2270.
Kalantari, A.S., and V.E. Cabrera. 2012. The effect of reproductive performance on the dairy cattle herd value assessed by integrating a daily dynamic programming model with a daily Markov chain model. Journal of Dairy Science. 95: 6160-6170.
Korver, S., van Arendonk, J.A.M. and W.J. Koops. 1985. A function for live weight change between two calvings in dairy cattle. Animal production. 40: 233-241.
Kristensen, A.R. 1988. Hierarchic Markov processes and their applications in replacement models. European Journal of Operational Research. 35: 207-215.
Kristensen, A.R. and I. Thysen. 1991. Ranking of dairy cows for replacement. Acta Agriculturae Scandinavica 41: 295-303.
Marsh, W., Dijkhuizen, A. and R. Morris. 1987. An economic comparison of four culling decision rules for reproductive failure in United States dairy herds using Dairyoracle. Journal of Dairy Science.70: 1274 -1280.
Meadows, C., Rajala-Schultz, P.J. and G.S. Frazer, 2005. A spreadsheet-based model demonstrating the nonuniform economic effects of varying reproductive performance in Ohio dairy herds. Journal of Dairy Science. 88: 1244-1254.
Olynk, N. J. and C.A. Wolf. 2009. Stochastic economic analysis of dairy cattle artificial insemination reproductive management programs. Journal of Dairy Science. 92:1290-1299.
Ostrovsky, I., 2010. Parallel programming in .NET 4. Coding Guidelines. Parallel computing platform group. Microsoft Corporation.
R Development Core Team. 2011. R: A language and environment for statistical computing. R Foundation

for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0. http://www.R-project.org/
Savicky, P., 2009. Spearman's rank correlation test. R package version 0.2-5. http://CRAN.R-project.org/package=pspearman.
Smith, B.J., 1973. Dynamic Programming of the Dairy Cow Replacement Problem. American Journal of Agricultural Economics. 55: 100-104.
van Arendonk, J. A. M. 1984. Studies on the replacement policies in dairy cattle. I. Evaluation on the replacement policies in dairy cattle. Animal Breeding and Genetics. 101: 330-340.
van Arendonk, J. A. M., 1985. Studies on the replacement policies in dairy cattle. II. Optimum policy and influence of changes in production and prices. Livestock Production Science. 13: 101-121.
van Arendonk, J.A.M. and A.A. Dijkhuizen. 1985. Studies on the replacement policies in dairy cattle. III. Influence of variation in reproduction and production. Livestock Production Science. 13: 333-349.
van Arendonk, J.A.M., 1988. Management guides for insemination and replacement decisions. Journal of Dairy Science. 71: 1050-1057.
VandeHaar, M., Bucholtz, H., Beverly, R., Emery, R., Allen, M., Sniffen, C. and R. Black. 1992. Spartan dairy ration evaluator/balancer. An agricultural integrated management software microcomputer program. CP-012 Version 2.01. CES, AES. Michigan State University, East Lansing.
Wood, P.D.P.. 1967. Algebraic model of the lactation curve in cattle. Nature. 216: 164-165